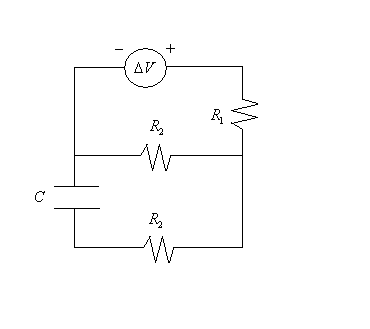
Resistor + Capacitors

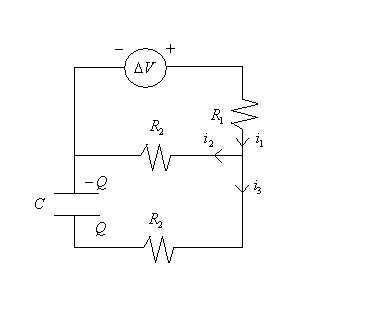
**Problem**

Start with example can do using equivalent resistance. For comparisons sake, we’ll take ΔV = 10, and R1 = 1, R2 = 2, and C = 5F. We’d like to know the currents in the circuit, and the charge on the capacitor, Q.

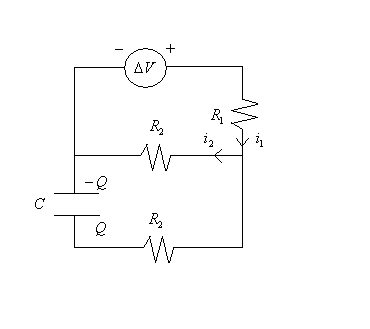


**Solution**

Like before, we label currents in the circuit, and this time the charge on the capacitor as well.



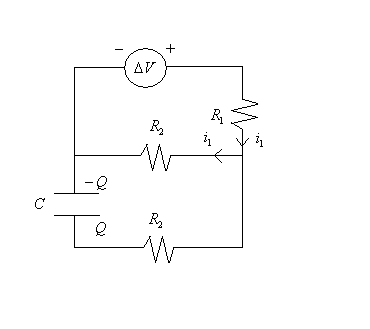
But with respect to the current, since we’re considering the capacitor to be charged up, there cannot be any current running through it. So i3 must be 0.



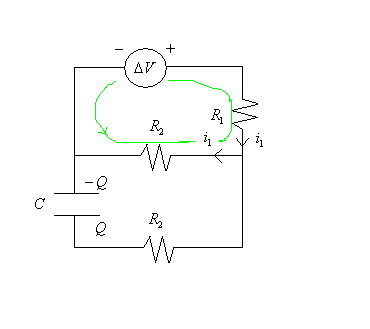
KCL then requires



Therefore we have,



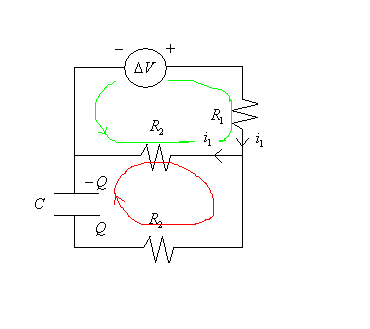
Next we make a closed loop around the circuit and apply KVL. Consider the following loop,



Then starting at the battery we have,



and now the final equation. We’ll choose another loop – the bottom half.

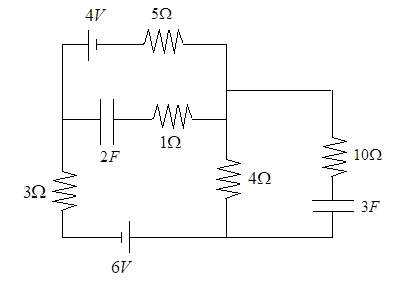


KVL applied to this loop, starting at the bottom of the capacitor is:



**Problem**

Consider the circuit below. Determine the current through the battery, the charge on the capacitors.



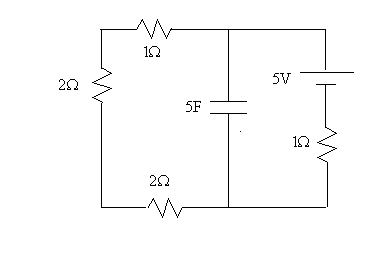
Current through the 3,5,4Ω resistors is I = ΔV/Req. = 10/12 = 0.83A. The charge on the 3F is therefore Q = CV = 3(5/6)∙4 = 10. The charge on the 2F would be Q = CV = 2(4-5∙10/12) = -0.33.

**Solution**

Current around perimeter is I = 10/15 = 0.67A. Charge on 3F capacitor is q = CV = (3)(10∙0.67) = 20C.

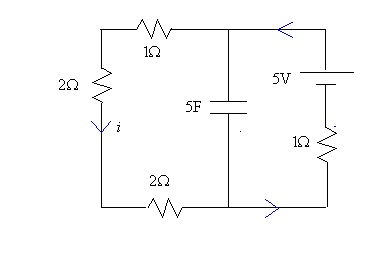
**Problem**

Consider the circuit below. What is the steady-state charge on the capacitor?



**Solution**

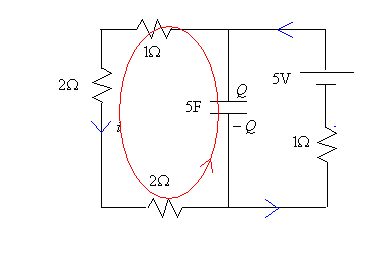
We can use KVL to answer this question. First recognize that the current running through the middle wire is zero (in the long run, i.e. steady state) since the capacitor is an open break in the circuit. Therefore current will just circulate around the outside of the circuit as illustrated below:



Using KVL around the outside loop we can determine what i is. We have, starting below the 5V battery and going counter clockwise:



Now then, assuming a charge Q and –Q on the capacitor plates, we can use KVL around the left loop to determine the charge:

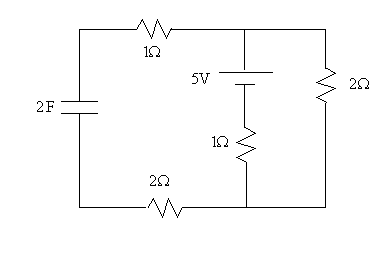


Starting below the capacitor…



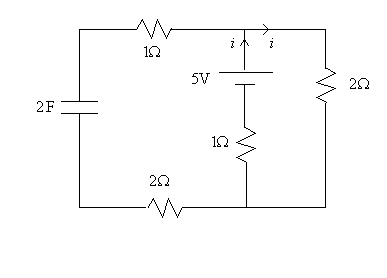
**Problem**

Consider the circuit below. What is the steady-state charge on the capacitor?

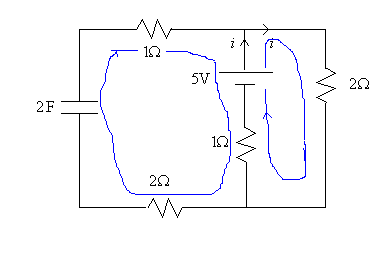


**Solution**

First note that the current going through the wire containing the capacitor is 0. Therefore the current looks like this,



Now we’ll use KVL twice along the following two arbitrary paths,



The left hand loop reads, starting from behind the capacitor



The right hand loop reads,

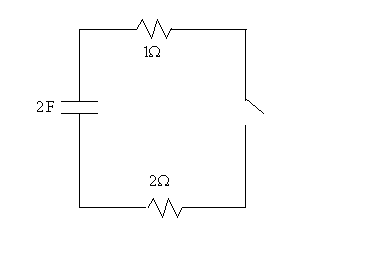


plugging this into the left hand loop equations we get,



**Problem**

Suppose there is 20C stored on the 2F capacitor below. When the switch is closed, what will be the current 1.2s later?

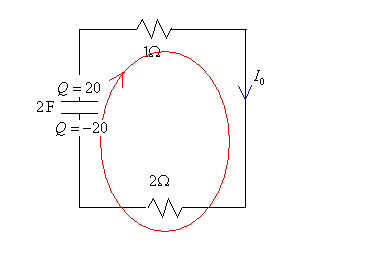


**Solution**

The current will follow the equation:



The initial current comes from KVL:





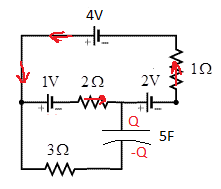
And finally, the time constant τ is given by τ = Req.Ceq. = (3)(2) = 6s. So the current as a function of time is:



Therefore at t = 1.2s the current is:

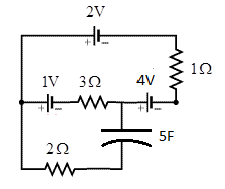


**Question 7. Might need to check!** For the circuit below, calculate the magnitude *and* direction of the current running through the 4V battery. And calculate the charge on the capacitor.



We know that no current will run through the capacitor in the long-time situation. Instead therefore, the current will simply run around the top loop. By Kirchoff’s law, going around the loop clockwise, it will be given by -4 +1i + 2 + 2i + 1 = 0 → i = 1/(3) = 0.33A. Then going around the bottom loop clockwise, and using Kirchoff’s voltage law we’ll have: -1 – 2(0.33) – Q/5 = 0 → Q = -1.67∙5 = -8.3 C.

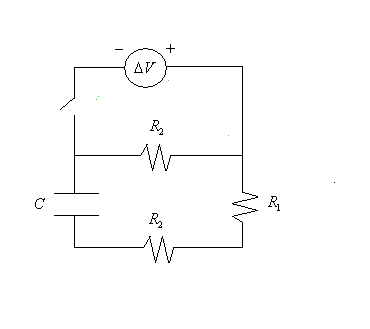
**Question 6.** For the circuit below, calculate the magnitude *and* direction of the current running through the 2V battery. And calculate the charge on the capacitor. You may assume that capacitor has been hooked up to the circuit for a long time.



The current running through the capacitor will be 0, and as such will just circulate around the top loop. Using KVL we have: -2V – I(1Ω) + 4V – I(3Ω) + 1V = 0 → -4I + 3 = 0 → I = 0.75A. Then using KVL around the lower loop we have: -1V + (3Ω)(0.75A) – Q/5F = 0 → 1.25V = Q/5F → Q = 6.25C.

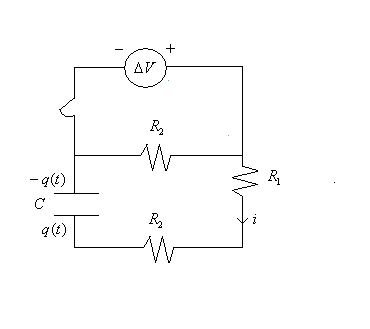
**Problem**

Suppose ΔV = 10V, R1 = 1Ω, R2 = 2Ω, and C = 5F. When we close the switch, how will the charge on the capacitor, current *i* through the capacitor, and current *i* through the top resistor behave as a function of time?



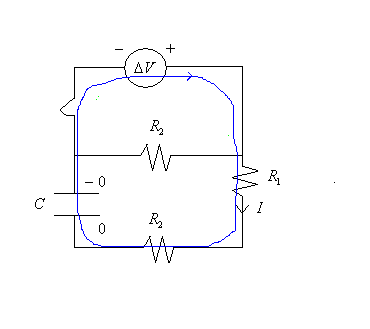
**Solution**

Well the charge on the capacitor will build up, and the current will decay accordingly. These will look like,





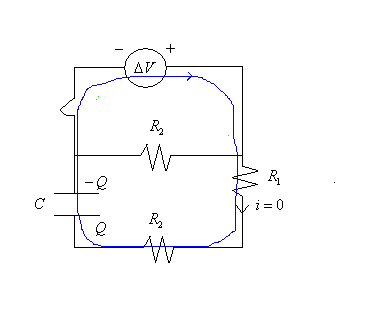
Q is the final charge on the capacitor, and I is the initial current. We can determine these by using KVL at the intial and final times. We’ll recognize that initially (t = 0), q = 0 as well,



Applying KVL.



At the final time (t = ∞), the capacitor is charged, and we have, using KVL along the blue loop,





The equivalent resistance through which the current runs is R = R1 + R2. So we have,

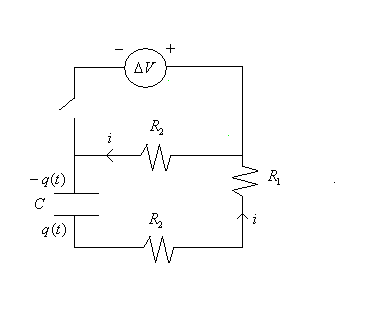


When will the current be equal to i = 1A? To answer we would solve,



**Problem**

Suppose now that we open the switch again. What will happen?

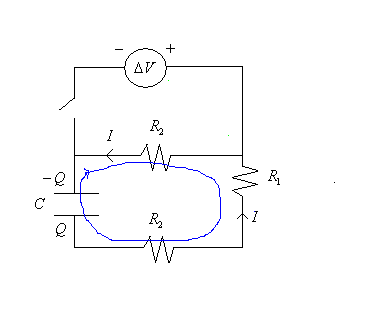


**Solution**

Then the capacitor will discharge through the middle wire. The charge and current will decay according to:



The initial charge will be as before, Q = 50. The initial current can be obtained by using KVL around the loop through which the current is flowing. We will remember that initially, q = Q so…



Applying KVL, we get,



and the effective resistance through which the current runs is:

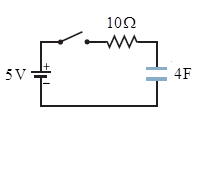


Therefore we’ll have,



**Problem**

For the circuit below, suppose that the capacitor is initially uncharged. After flipping the switch, what will be the final charge on the capacitor? When will the charge on the capacitor be equal to 1/3 its final value?



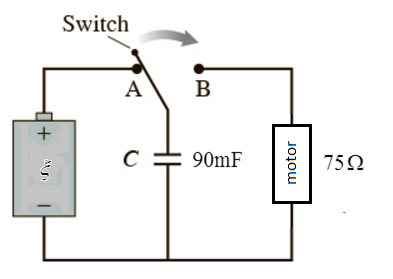
**Solution**

The final charge will be Q = CV = 20 Coulombs. The charge as a function of time is:

q(t) = Q(1-e-t/RC). It will be 1/3 final value when



**Question 6.** We want to use a capacitor to run a motor to raise a 3kg block 2m above the ground in 0.45s. So we charge a capacitor C = 90mF using a battery with potential difference ξ, and then allow it to discharge through the motor (by flipping the switch to B). If we can model the motor as having a resistance of R = 75Ω, then what should ξ be?

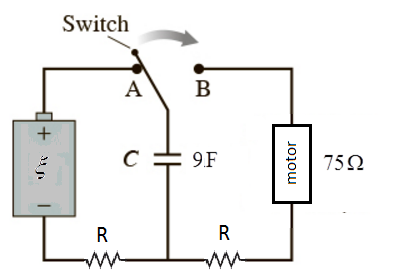


The energy required to raise the weight would be: PE = mgh = (3)(9.8)(2) = 58.8J. The energy stored on the capacitor at any time t is PE = (1/2)CΔV2, where ΔV = ΔVie-t/τ = ξe-t/RC. And so the energy delivered to the motor by time t is PE = (1/2)C(ΔV0)2 – (1/2)C(ΔV)2 = (1/2)C[ξ2 – ξ2e-2t/RC] = (1/2)Cξ2[1-e-2t/RC]. And we need this energy to be 58.8J by t = 0.45s, so we must solve:



**Question 6.** For each part, remember current will not run through an open break….(a) Suppose that the 9F capacitor in the diagram is charged with an ξ = 150V battery, in series with an R = 25Ω resistor, while the switch is in position A. How long until the capacitor’s potential difference reaches 120V?

(b) Now suppose the motor can be approximated as a 75Ω resistor, and is in series with another 25Ω resistor. If the motor must have a potential difference of at least 30V across it to run, how long (in minutes) will it do so after the switch is flipped to position B?



The voltage across the capacitor will be given by:



and so the current in the circuit will be given by:



And the voltage across the motor will be given by:



and so we must solve:



**Question 8.** A 200μF defibrillator capacitor is charged to 1200V. When fired through a patient’s chest, it loses 90% of its charge in 40ms. What is the resistance of the patient’s chest?

The charge on the capacitor is given by:



and it drops to 10% charge in time t = 40ms. So we have:

